MATHS MATTERS

In primary school the aim is that children use mental methods whenever possible but for calculations that they cannot do in their heads they can use an efficient written method or calculator.

The written methods support and build on a range of mental strategies that the children have developed throughout their time at primary school.

Please note that not all children will progress through each stage and it is important that children are not taught a method too early without the understanding of why the method works and when to use it appropriately. This is also why the methods are given in steps rather than year groups.

When referring to mental methods this does not necessarily mean calculations performed exclusively 'in the head' but also means children may do jottings or informal calculations to support their working.

Top Tips to help and support your child with maths

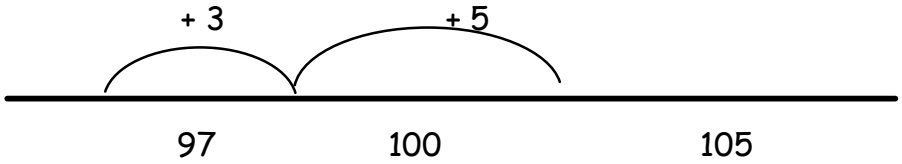
- Mental maths is the key to success and time spent practising number facts and multiplication tables is essential.
- There are not as many tables to learn as you think! To learn your tables up to 10 x10, that's a maximum of 50 sums to learn, not 100 because 4 x 3 is the same as 3 x 4. Then cross 10 more off for the 1-times table, which only leaves 40 sums you have to learn. If you learn the 2 and 10 times table then you are left with just 28 sums to learn. That sounds more manageable.
- Other important skills are doubling and halving, and counting on and back in ones and multiples of ten and one hundred.
- Make maths part of your everyday life. On the way to school, in the car, while you're cooking ask your child sums.
- Whenever you see numbers, turn them into simple sums. Add up numbers on a car number-plate. Or in the supermarket, add up weights on cereal packets.
- Have fun. There are lots of mathematical games and logic puzzles available.
- Don't ask all the questions. Children love to test adults as well!

Glossary of mathematical terms

Basic operations:

Symbol	Words used
+	add, addition, sum, plus, increase, total
−	subtract, subtraction, minus, less, difference, decrease, take away
x	multiply, multiplication, product, by, times, lots of
÷	divide, division, goes into, how many times

Bridging: when children cross a boundary e.g. multiples of 10, 100, 1000. We refer to this as bridging e.g. adding 8 onto 97 children will add 3 to 100 then add 5 (the children have partitioned 8 into 3 and 5 and used 100 as a bridge).



Divisor: the number you divide by e.g. in 15 ÷ 3 = 5, the 3 is the divisor.

Estimate: (to make) an approximate or rough calculation, often based on rounding e.g. 32 x 58 is approximate 30 x 60 (rounding to the nearest 10) = 1800

Inverse operations:
Addition and subtraction are inverse operations e.g. 3 + 6 = 9 *inverse* 9 – 6 = 3
Multiplication and division are inverse operations e.g. 5 x 6 = 30 *inverse* 30 ÷ 6 = 5

Multiple:

- the multiples of 10 are 10, 20, 30, 40 …
- the multiples of 2 are 2, 4, 6, 8, 10, 12 …
- the multiples of 7 are 7, 14, 21, 28, 35 …

Partition: to partition a number means, 'breaking the number up in different ways.' The most common way to partition in primary school is into hundreds, tens and ones e.g. 472 = 400 + 70 + 2.
but numbers can also be partitioned in different ways
e.g. 8 = 7 + 1, 6 + 2, 5 + 3, 4 + 4.

Place value: the value of a digit depending on its place in a number
e.g. 354 the value of the 4 is four ones*, whereas in the number 435 the 4 has a value of four hundreds.
*Please note that we no longer use the term 'units' but say 'ones' instead.

Product: the result of multiplying a set of numbers e.g.
The product of 6 and 3 is 18.

Single-digit numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are single-digit numbers.

Two-digit numbers: 10, 26, 84, 60, 32, 77 etc. are some examples of two-digit numbers.

Whole numbers: the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 etc.

Addition

To add successfully, children need to be able to:

- recall all addition pairs to 9 + 9 e.g. 3 + 5, 1 + 6, 8 + 4, 7 + 9
- recall all number bonds to 10 e.g. 4 + 6, 7 + 3
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition (breaking up) two-digit and three-digit numbers into multiples of 100, 10 and 1 e.g. 346 = 300 + 40 + 6

Written methods for addition

Stage 1: The empty number line

The empty number line helps to record the steps on the way to calculating the total.

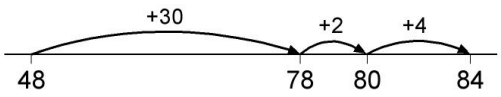
Counting on bridging over 10

8 + 7

8 + 7 = 15

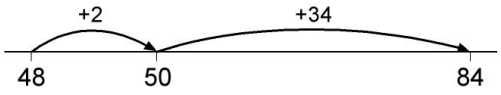
Counting on in multiples of 100, 10, 1

48 + 36



48 + 36 = 84

or:



Stage 2: Partitioning

The next stage is to record <u>mental methods</u> using partitioning.

Record steps in addition using partitioning

47 + 76 = 47 + 70 + 6 = 117 + 6 = 123

or

47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123

Note that the children's recording of this method may be more informal (jottings) and they may only record the steps towards the final answer e.g.

36 + 48 = 70 + 14 = 84

(the calculation of 30 + 40 and 6 + 8 being done 'in the head')

Stage 2: Partitioning continued

Partitioned numbers are then written under one another. This mirrors the column method and also links mental methods.

$$\begin{array}{rcl} 47 & = & 40 + 7 \\ +\,76 & & 70 + 6 \\ \hline & & 110 + 13 = 123 \end{array}$$

Stage 3: Expanded method in columns

Write the numbers in columns.

Adding the most significant digit first (as this compliments mental strategies)

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 110 \\ 13 \\ \hline 123 \end{array}$$

- The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'.

Adding the ones first (once children have gained confidence)

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 13 \\ 110 \\ \hline 123 \end{array}$$

Stage 4: Column method

Carried digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', **not** 'carry one'.

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 123 \\ {\scriptstyle 11} \end{array} \qquad \begin{array}{r} 258 \\ +\ 87 \\ \hline 345 \\ {\scriptstyle 11} \end{array} \qquad \begin{array}{r} 366 \\ +458 \\ \hline 824 \\ {\scriptstyle 11} \end{array}$$

Both the expanded method and column addition can be used with larger numbers and decimals.

Subtraction

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20 e.g. 6 + 7 = 13, 15 – 8 = 7
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact, 16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Written methods for subtraction

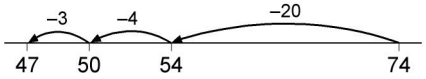
Stage 1: Using the empty number line

Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

15 – 7

15 – 7 = 8

74 – 27



74 – 27 = 47 worked by counting back

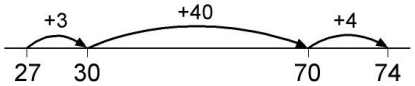
The steps may be recorded in a different order:

The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (see below). With practice, children will need to record less information and decide whether to count back or forward. Choosing the more efficient strategy for calculations such as;

57 – 12, 304 – 9 (count back), 86 – 77, 321 – 134 (count up).

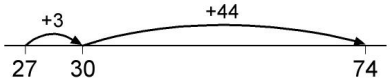
The counting-up method

74 – 27

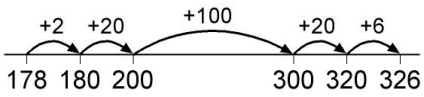


74 – 27 = 47 *(by adding the steps jumped).*

or:

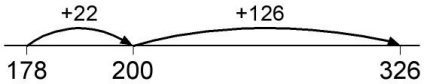


326 – 178



326 – 178 = 148

Or more efficiently using less steps:



The counting-up method may be the main method used by some children throughout juniors or used as an alternative method when dealing with subtractions with zeros such as 1300 – 1156.

The counting-up method can be used with larger numbers, decimals and differences in time but may become less efficient than column subtraction.

Stage 2/3: Expanded layout, leading to column method

This method would be supported with practical apparatus. Once confident this then leads to the column method. The amount of time spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Example: **563 – 241**

Expanded method *leading to*

$$\begin{array}{r} 500 + 60 + 3 \\ -\ 200 + 40 + 1 \\ \hline 300 + 20 + 2 \end{array} \qquad \begin{array}{r} 563 \\ -\ 241 \\ \hline 322 \end{array}$$

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Example: **74 – 27**

→ *column method*

$$\begin{array}{r} 70 + 4 \\ -\ 20 + 7 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 60}\ \ {\scriptstyle 14} \\ \cancel{70} + \cancel{4} \\ -\ 20 + 7 \\ \hline 40 + 7 \end{array} \qquad \begin{array}{r} {\scriptstyle 6}\ {\scriptstyle 14} \\ \cancel{7}\ \cancel{4} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$$

For the column method (decomposition) start from the right-hand column. In this method 4 minus 7 cannot be done (do not swap the digits over!). So exchange a ten for ten ones. This leaves 6 tens and 14 ones. In effect, what has been done is to partition the 74 into 60 + 14, which is the same as in the expanded method but more efficiently recorded.

Example: **741 – 367**

→ *column method*

$$\begin{array}{r} 700 + 40 + 1 \\ -\ 300 + 60 + 7 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 600}\ \ {\scriptstyle 130}\ \ {\scriptstyle 11} \\ \cancel{700} + \cancel{40} + \cancel{1} \\ -\ 300 + 60 + 7 \\ \hline 300 + 70 + 4 \end{array} \qquad \begin{array}{r} {\scriptstyle 6}\ {\scriptstyle 13}\ {\scriptstyle 11} \\ \cancel{7}\ \cancel{4}\ \cancel{1} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$$

Example: **563 – 271**, partitioning the hundreds, or adjustment from the hundreds to the tens.

or → *column method*

$$\begin{array}{r} 500 + 60 + 3 \\ -\ 200 + 70 + 1 \\ \hline \end{array} \qquad \begin{array}{r} 400 + 160 + 3 \\ -\ 200 + \ 70 + 1 \\ \hline 200 + \ 90 + 2 \end{array} \qquad \begin{array}{r} {\scriptstyle 400}\ \ {\scriptstyle 160} \\ \cancel{500} + \cancel{60} + 3 \\ -\ 200 + 70 + 1 \\ \hline 200 + 90 + 2 \end{array} \qquad \begin{array}{r} {\scriptstyle 4}\ {\scriptstyle 16} \\ \cancel{5}\ \cancel{6}\ 3 \\ -\ 2\ 7\ 1 \\ \hline 2\ 9\ 2 \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes '160 minus 70'.

Division

To divide successfully in their heads, children need to be able to:

- recall multiplication and division facts to 10 × 10 e.g. 4 x 7, 56 ÷ 8, 5 x 9, 14 ÷ 2
- recognise multiples of one-digit numbers e.g. 49 is a multiple of 7
- divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value e.g. 12 ÷ 3 = 4 *so* 120 ÷ 3 = 40.
- know how to find a remainder working mentally e.g. find the remainder when 48 is divided by 5;

  'I know 9 x 5 = 45 therefore 48 ÷ 5 = 9 remainder 3'
- understand and use multiplication and division as inverse operations.

  6 x 7 = 42 inverse 42 ÷ 7 = 6

To carry out written methods of division successfully, children need to be able to:

- understand division as repeated subtraction; 15 ÷ 5 = 15 – 5 – 5 – 5 (therefore we have taken away 3 lots of 5) so 15 ÷ 5 = 3
- estimate how many times one number divides into another e.g. how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

Written method for division

Stage 1: Chunking method (using multiples and known facts)

This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks.

For 81 ÷ 3

	27	
3	81	
-	30	**10** lots of 3
	51	
-	30	**10** lots of 3
	21	
-	21	**7** lots of 3

Ans. **27** *(lots of 3)*

For the sum 81 ÷ 3 the divisor is 3. So children subtract multiples of 3.

This method allows children to be successful with limited times tables knowledge (x10, x5, x2) although this will require more steps, 'chunks.'

For 72 ÷ 5

	14	**remainder 2**
5	72	
-	50	**10** x 5
	22	
-	20	**4** x 5
	2	

Stage 2: Chunking method (using largest possible multiples)

As children gain confidence they can refine this method and reduce recording by using the largest possible multiples. Children need to be confident with multiplication facts and understanding of place value.

For 81 ÷ 3

```
   27
3 | 81
- 60    20 x 3
  21
- 21     7 x 3
```

For 196 ÷ 6

```
   32  remainder 4
6 | 196
- 180   30 x 6
  16
- 12     2 x 6
   4
```

Stage 3: Short division ÷ by a single digit

Short division can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.

For 81 ÷ 3

$$3\overline{)8\,{}^{2}1}\quad 2\,7$$

For 291 ÷ 3

$$3\overline{)2\,9\,{}^{2}1}\quad 9\,7$$

Stage 4: Long division ÷ by a two digit number (chunking method)

For 560 ÷ 24

```
   23  rem. 8
24 | 560
- 480   20 x 24
  80
- 72     3 x 24
   8
```

Multiplication

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10 × 10;
- partition numbers into multiples of one hundred, ten and one; 354 = 300 + 50 + 4
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see addition).

Written methods for multiplication

Stage 1: The grid method

This method links to mental methods. Children are encouraged to begin with an estimation.

38 × 7 =

x	30	8
7	210	56

210 + 56 = 266

Children will need to use their tables and place value knowledge e.g.
'I know 3 x 7 = 21 so 30 x 7 = 210'

An alternative way to record which supports adding the products in columns.

x	7
30	210
8	56
	266

The grid method may be the main method used by some children throughout KS2 and can be used with larger numbers and decimals.

Stage 2: Expanded short multiplication

The next step is to represent the method of recording in a column format, but showing the working.

$$\begin{array}{r} 30+8 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array} \qquad \begin{array}{l} 30\times 7=210 \\ 8\times 7=\ \ 56 \end{array} \qquad \textit{leading to} \qquad \begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array}$$

Please note the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven' although the children should use their tables and place value knowledge e.g.

'I know 3 x 7 = 21 so 30 x 7 = 210'

Stage 3: Short multiplication

The carry digits are recorded below the line.

$$\begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 266 \\ \scriptstyle 5 \end{array}$$

Children need to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage e.g. this calculation involves adding 210 and 50 mentally.

Stage 4: Grid method (Two-digit by two-digit products)

Children should be encouraged to estimate first.

56 × 27 is approximately 60 × 30 = 1800 (using known fact 6 x 3 = 18)

x	20	7
50	1000	350
6	120	42

$$\begin{array}{r} =1350 \\ = \\ 162 \\ \hline 1512 \\ 1 \end{array}$$

Some children may be taught the compact long multiplication method for two-digit multiplied by two-digit by the end of KS2.